



Progetto S5 - “Definizione dell'input sismico sulla base degli spostamenti attesi”

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Performance-Based Seismic Design

- Requires accurate prediction of the structural performance
- Displacement/deformation is directly related to damage, hence displacement-based approaches are preferred to force-based ones

DBD METHODS: CLASSIFICATION

- A. Equivalent linearization procedures that use overdamped spectra
 - Rosenblueth and Herrera;
 - Güllkan and Sozen;
 - Iwan;
 - Capacity Spectrum Method (Freeman)
 - Priestley and Kowalsky (DDBD);
 - ATC-40 (CSM);

- B. Equivalent linearization procedures that use inelastic spectra or “displacement coefficients”
 - Fajfar (N2-method);
 - Improved CSM (Chopra)

- C. Displacement coefficient methods:
 - Newmark and Hall;
 - Miranda;
 - FEMA 356

A. Equivalent linearization procedures

Use an equivalent elastic SDOF system (substitute system) to estimate the maximum displacement of the nonlinear system

Inelastic SDOF defined from the pushover curve

$$\ddot{x} + 2\xi_0\omega_0\dot{x} + \frac{F(x)}{m} = -\ddot{x}_g$$

Elastic SDOF

- Equivalent period (T_{eq})
- Equivalent viscous damping ratio (ξ_{eq})
(equal energy principle, Jacobsen)



Defined from:
hysteretic properties
and ductility
of the nonlinear system

Equivalent elastic SDOF

$$\ddot{x}_{eq} + 2\xi_{eq}\omega_{eq}\dot{x}_{eq} + \omega_{eq}^2 x_{eq} = -\ddot{x}_g$$

DEFINITION OF THE EQUIVALENT SDOF SYSTEM

Rosenblueth and Herrera (Geometric Method, 1964)

T_{eq} : based on secant stiffness at maximum deformation (K_s)

$$\omega_{eq} = \sqrt{\frac{k_s}{m}} = \frac{2\pi}{T_{eq}}$$

ξ_{eq} : equal energy principle at steady-state harmonic response
(hysteretic energy)

For a bilinear model with initial stiffness k_0 and post yield stiffness αk_0 :

$$\frac{T_{eq}}{T} = \sqrt{\frac{k_0}{k_s}} = \sqrt{\frac{\mu}{1 - \alpha + \alpha\mu}}$$

$$\xi_{eq} = \xi_0 + \frac{2}{\pi} \left[\frac{(1 - \alpha)(\mu - 1)}{\mu - \alpha\mu + \alpha\mu^2} \right]$$

Gülkan and Sozen (1974)

ξ_{eq} : Harmonic loading assumption leads to an overestimation of the damping

Proposed empirical relation for Takeda hysteretic model:

$$\xi_{eq} = \xi_0 + 0.2 \left(1 - \frac{1}{\sqrt{\mu}} \right)$$

(Method extended to MDOF systems by replacing the displacement ductility by a damage ratio).

Iwan (1980)

Empirical equations:

$$\frac{T_{eq}}{T} = 1 + 0.121(\mu - 1)^{0.939}$$

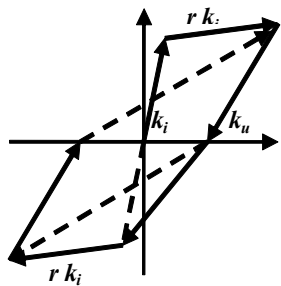
$$\xi_{eq} = \xi_0 + 0.0587(\mu - 1)^{0.371}$$

Kowalsky (1994)

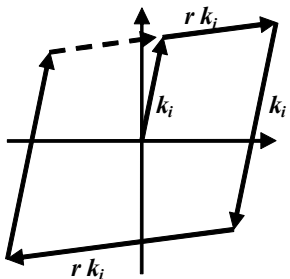
T_{eq} : based on secant stiffness at maximum deformation (K_s)

ξ_{eq} : equal energy principle at steady-state harmonic response.

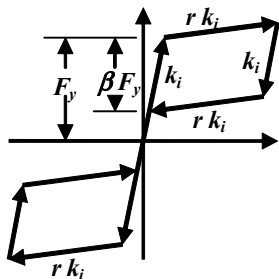
Values depends on the hysteretic model (hysteretic energy):



Modified Take

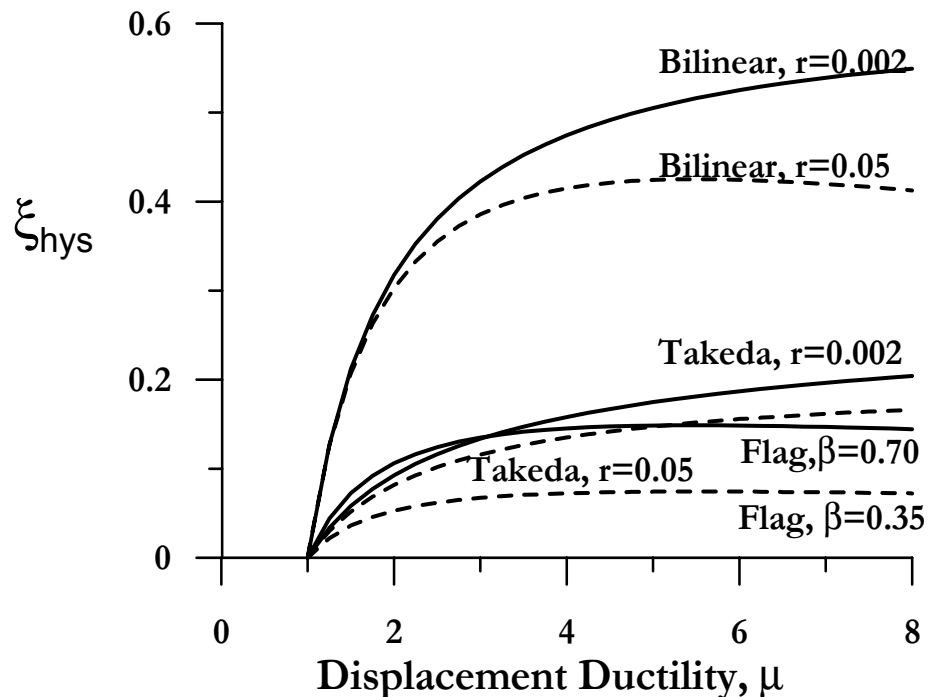


Bilinear



Flag-shaped

$$\xi_{eq} = \xi_0 + \xi_{hys}$$



DDBD for MDOF systems

Priestley and Kowalsky (2000)

- Select the design displacement - (Δ_d)
- Define the expected displacement shaped of the structure (empirical expression for different structural typologies) - (Δ_i)
- Define the design displacement for the equivalent SDOF system

$$\Delta_{sys} = \frac{\sum m_i \Delta_i^2}{\sum m_i \Delta_i}$$

- Define the effective mass of the SDOF system

$$M_{sys} = \frac{\sum m_i \Delta_i}{\Delta_{sys}}$$

- Estimate the ductility of the structure (from material and member properties) and calculate the equivalent viscous damping of the SDOF system

$$\mu = \Delta_d / \Delta_y$$

$$\xi = \xi(\mu, \text{structural type})$$

- Define the damped response spectrum

$$S_d(T, \xi) = S_d(T, 5\%) * \left(\frac{10}{2 + \xi} \right)$$

- Find, from the design displacement, the effective period (and stiffness)

$$k_{eff} = \frac{4\pi^2 M_{sys}}{T_{eff}^2}$$

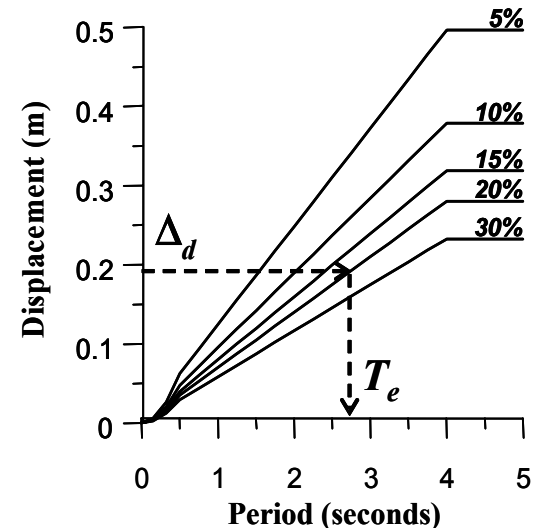
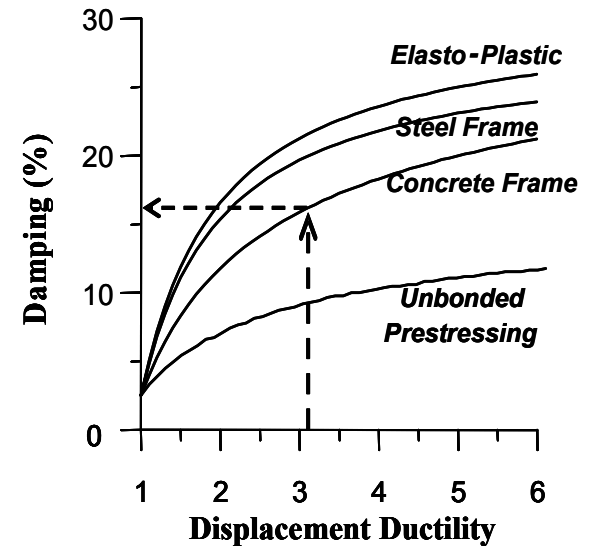
- Compute the base shear

$$V_b = k_{eff} \Delta_{sys}$$

- Find the correspondent design storey forces

$$F_i = \left(\frac{m_i \Delta_i}{\sum m_i \Delta_i} \right) V_b$$

- Design the structural members

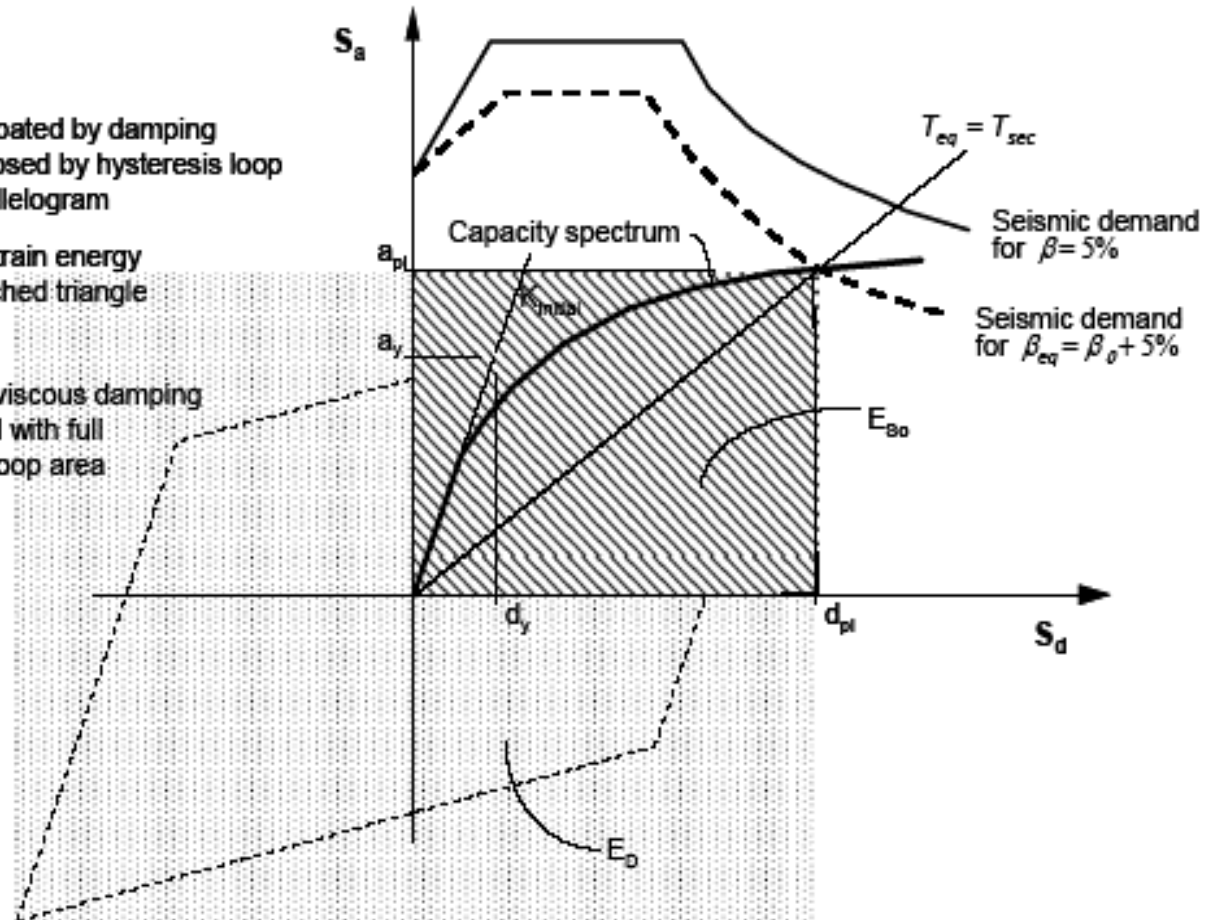


Capacity Spectrum Method (CSM) (ATC-40)

E_D = Energy dissipated by damping
 = Area enclosed by hysteresis loop
 = Area of parallelogram

E_{so} = Maximum strain energy
 = Area of hatched triangle
 = $a_{pl} d_{pl} / 2$

β_0 = Equivalent viscous damping
 associated with full
 hysteresis loop area
 $= \frac{1}{4\pi} \frac{E_D}{E_{so}}$



Capacity Spectrum Method (CSM) (ATC-40)

- Define the pushover curve and convert it into the Acceleration vs. Displacement format (Capacity Curve) using the first mode dynamic properties

$$S_a = \frac{V_{base}}{\Gamma_1 M_1}$$

$$S_d = \frac{\Delta_{control-node}}{\Gamma_1 \Phi_{control-node,1}}$$




- Plot the design spectrum in ADRS format
- Find K_s at the interception (design point)
- Equivalent damping (β_{eq}): area enclosed by the capacity curve at design point
→

Equations

$$T_{eq} = T_0 \sqrt{\frac{\mu}{1 + \alpha\mu - \alpha}}$$

$$\beta_{eq} = 0.05 + k \frac{2(\mu - 1)(1 - \alpha)}{\pi\mu(1 + \alpha\mu - \alpha)}$$

k: factor accounting for changes in the hysteretic behavior of RC structures

Hysteretic Behavior	β_0	κ	
Type A	≤ 0.1625	1.0	 Full hysteretic loop
	> 0.1625	$1.13 - 0.51 \times (\pi/2) \times \beta_0$	
Type B	≤ 0.25	0.67	 Degraded structure
	> 0.25	$0.845 - 0.446 \times (\pi/2) \times \beta_0$	
Type C	Any value	0.33	 Intermediate response

Definition of the inelastic spectrum (scaling factors for ADRS spectrum):

$$SR_a = \frac{3.21 - 0.68 \ln(100\beta_{eff})}{2.12} \quad \longrightarrow \quad \text{Constant acceleration portion of the spectrum}$$

$$SR_v = \frac{2.31 - 0.41 \ln(100\beta_{eff})}{1.65} \quad \longrightarrow \quad \text{Constant velocity portion of the spectrum}$$

Capacity Spectrum Method (CSM) (FEMA 440 updates)

- ATC-40 procedure overestimate the response for low period systems
- Developement of new expressions for T_{eq} , β_{eq}
- Ridefinition of the overall procedure
- Introduction of an additional scaling factor M in order to maintain the same graphical representation of the previous method

CSM: Proposed formulation

- Select a trial value for the equivalent damping (β_{tr}) \longrightarrow define $ADRS(\beta_{tr})$
 - Find a first design point (d_{pi}, a_{pi})
 - Define the bilinear capacity curve of the equivalent system (α, μ)
 - Find the correspondent couple of effective period and damping $\left(\frac{T_{eff}}{T_0}, \beta_{eff} \right)$

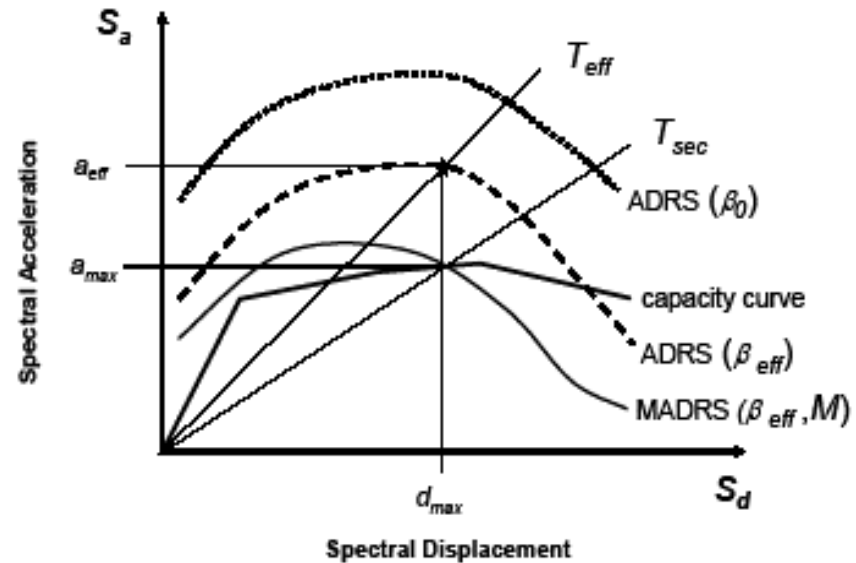
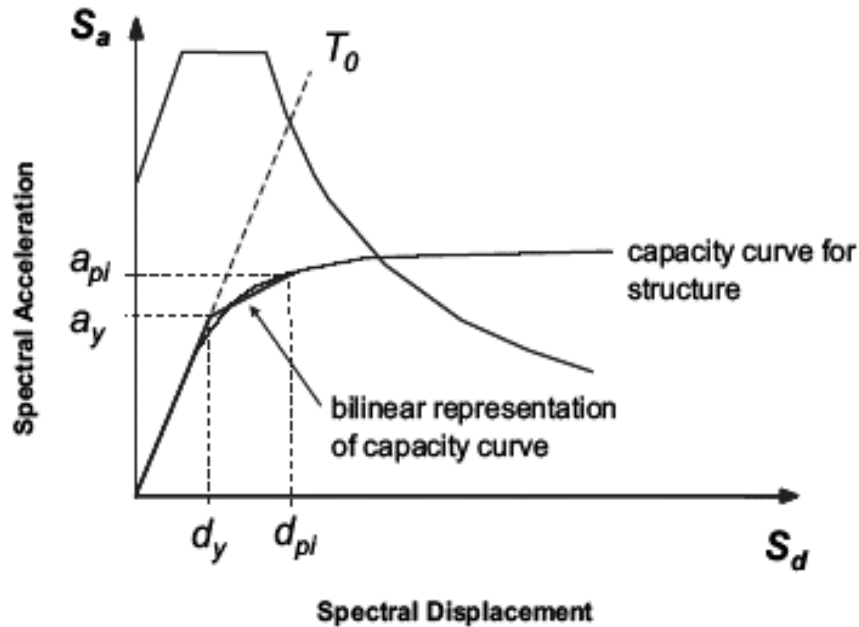
$$M = \frac{a_{pi}}{a_{max}} = \left(\frac{T_{sec}}{T_{eff}} \right)^2 = \left(\frac{T_{sec}}{T_0} \right)^2 \left(\frac{T_{eff}}{T_{sec}} \right)^2 \beta_{eff}$$
 - Define $ADRS(\beta_{eff})$
 - Scale the ADRS ordinates by M

{

Where:

$$\frac{T_{sec}}{T_0} = \sqrt{\frac{\mu}{1 + \alpha(\mu - 1)}}$$
- \longrightarrow
- Find the performance point compare with d_{pi}

Capacity Spectrum Method (CSM) (FEMA 440)



Equations: effective period and equivalent damping are function of:

- Displacement ductility (μ)
- Hysteretic model
- post elastic stiffness (α)

For $\mu < 4.0$:

$$T_{\text{eff}} = \left[G(\mu - 1)^2 + H(\mu - 1)^3 + 1 \right] T_0$$

$$\beta_{\text{eff}} = A(\mu - 1)^2 + B(\mu - 1)^3 + \beta_0$$

For $4.0 \leq \mu \leq 6.5$:

$$T_{\text{eff}} = \left[I + J(\mu - 1) + 1 \right] T_0$$

$$\beta_{\text{eff}} = C + D(\mu - 1) + \beta_0$$

For $\mu > 6.5$:

$$T_{\text{eff}} = \left\{ K \left[\sqrt{\frac{(\mu - 1)}{1 + L(\mu - 2)}} - 1 \right] + 1 \right\} T_0$$

$$\beta_{\text{eff}} = E \left[\frac{F(\mu - 1) - 1}{F(\mu - 1)^2} \right] \left(\frac{T_{\text{eff}}}{T_0} \right)^2 + \beta_0$$

NOTE: coefficients A to L (tabulated) have been optimized for SDOF oscillator and not actual buildings

N2 - Method

Fajfar (1999)

It is a variant of the CSM, based on *inelastic spectra*:

$$S_a = \frac{S_{ae}}{R_\mu}$$

$$S_d = \frac{\mu}{R_\mu} S_{de} = \frac{\mu}{R_\mu} \frac{T^2}{4\pi^2} S_{ae} = \mu \frac{T^2}{4\pi^2} S_a$$

Where:

$$R_\mu = (\mu - 1) \frac{T}{T_0} + 1, \quad T \leq T_0$$

$$R_\mu = \mu, \quad T \geq T_0$$



Equal displacement rule

$$T_0 = 0.65 \mu^{0.3} T_c \leq T_c$$



From Vidic et al. (1994)

T_c : characteristic period of the ground motion: separates constant acceleration and velocity portions;

T_0 : transition period. A simplified method might be used assuming $T_0 = T_c$

Procedure:

- Perform a pushover analysis: storey forces proportional to mass and an assumed displacement shape
- Transform the pushover curve of the structure into the Force-Displacement relationship of the equivalent SDOF system

$$Q_{MDOF} = \Gamma Q_{SDOF} = \Gamma Q^* \quad (\text{Where } Q \text{ refers to all quantities of interest})$$

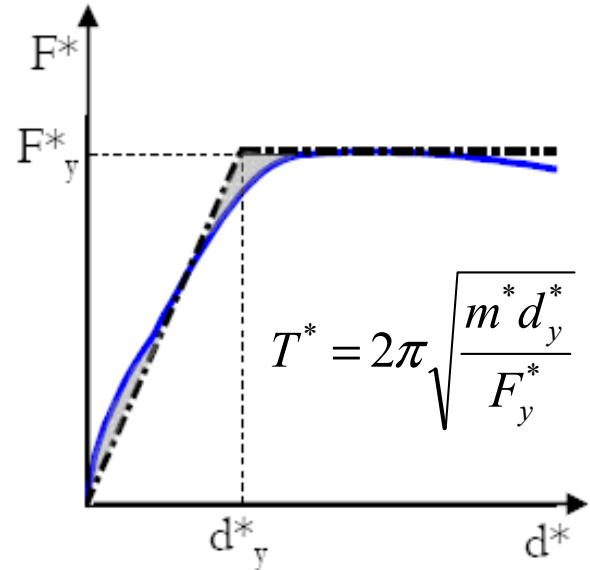
$$\Gamma = \frac{\sum m_i \Phi_i}{\sum m_i \Phi_i^2}$$

- Define the Elasto-Plastic idealization
- Determine the seismic demand for the SDOF system:

$$T^* \geq T_0 \longrightarrow d_{\max}^* = S_d(T^*)$$

$$T^* < T_0 \longrightarrow R_\mu = \frac{S_{ae}}{S_{ay}} = \frac{F_y / m^*}{S_{ay}} = \frac{F_y / \sum m_i \Phi_i}{S_{ay}}$$

$$\longrightarrow d_{\max}^* = \mu \cdot d_y^*$$



- Check the expected maximum displacements of the MDOF system $\Delta_i = \Phi_i \Gamma d_{\max}^*$

Improved CSM (Chopra, 1999)

ATC-40

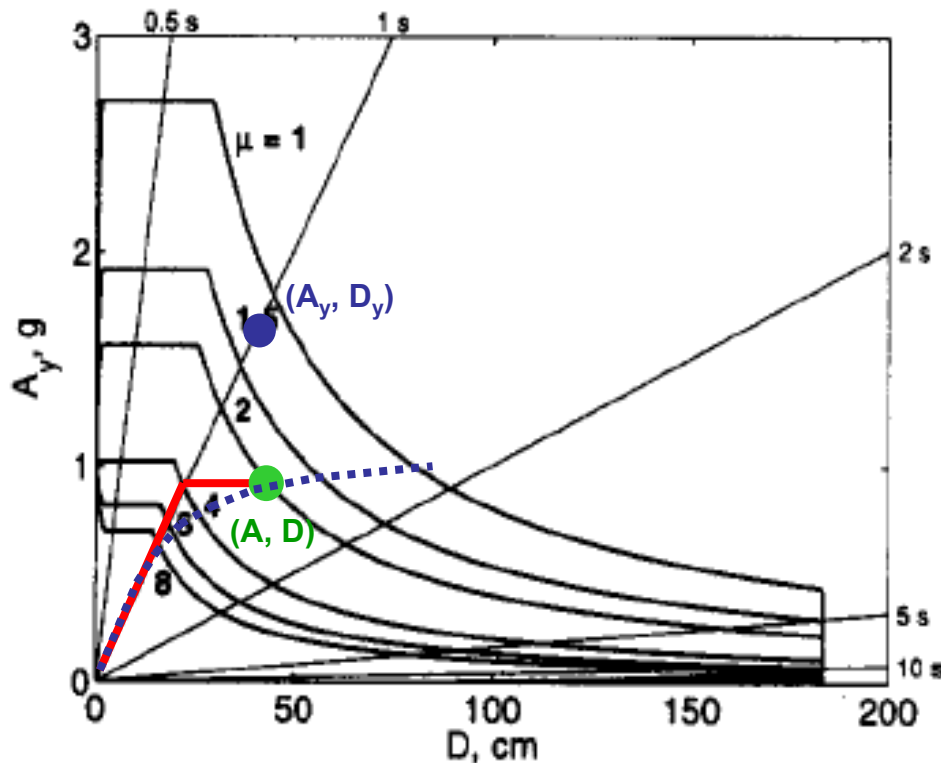
- Elastic design spectra cannot be used to estimate the peak displacement demand of inelastic systems
- The ATC-40 procedure leads to inconsistent results in the velocity- and displacement-sensitive regions of the spectrum
(the equal displacement rule holds)

Proposed methodology

- Deformation demand still determined at the intersection of the capacity diagram and the demand diagram
- The constant ductility demand diagram for inelastic system is adopted (inelastic spectra)

Improved CSM: Proposed formulation

- Define the inelastic design spectra for several values of ductility (μ)
- Plot the design spectra and the capacity curve of the system on the same plot (acceleration-displacement format)
- Find the design displacement (D) at the interception point where the ductility factor calculated from the capacity diagram matches the ductility value associated with the intersecting demand curve



$$D = \mu \frac{1}{R} \left(\frac{T_n}{2\pi} \right)^2 A$$

R- μ -T relationships from
Newmark and Hall (1982),
Krawinkler and Nassar (1992),
Vidic et al. (1994)
or others.

C. Coefficient Methods

Maximum displacement of the nonlinear system estimated from the maximum deformation of the linear elastic SDOF system with the same stiffness and damping adopting *displacement modification factors*

$$\Delta_i = C\Delta_e$$

Newmark and Hall (1982)

$$C = \mu, \quad T < T_a = 1/33 \text{ s}$$

$$C = \frac{\mu}{(2\mu - 1)^\beta}, \quad T_a \leq T < T_b = 0.125 \text{ s}$$

$$C = \frac{\mu}{\sqrt{2\mu - 1}}, \quad T_b \leq T < T_c'$$

$$C = \frac{T_c}{T}, \quad T_c' \leq T < T_c$$

$$C = 1, \quad T \geq T_c$$

where:

T_a, T_b, T_c are spectrum properties and

$$\beta = \frac{\log(T/T_a)}{2 \log(T_b/T_a)}$$

$$T_c' = \frac{\sqrt{2\mu - 1}}{\mu} T_c$$

Miranda

Proposed empirical equations from statistical analysis
of mean inelastic displacement ratio

For firm soil conditions
(Miranda, 2000)

$$C = \left[1 + \left(\frac{1}{\mu} - 1 \right) \exp(-12 T \mu^{-0.8}) \right]^{-1}$$

For soft soil conditions
(García and Miranda, 2004)

$$\begin{aligned} \tilde{C}_\mu = 1 + (\mu - 1) & \left[\theta_1 + \theta_2 \left(\frac{T}{T_g} \right)^{-4.2} \right] \\ & + \theta_3 (\mu - 1)^{0.5} \left(\frac{T_g}{T} \right) \exp \left[\left(2.3 - \frac{32}{\mu} \right) \left(\ln \left\{ \frac{T}{T_g} \right\} - 0.1 \right)^2 \right] \\ & - 0.08 \left(\frac{T_g}{T} \right) \exp \left[-70 \left(\ln \left(\frac{T}{T_g} + 0.67 \right) \right)^2 \right] \end{aligned}$$

where, T is the period of vibration, T_g is the predominant period of the ground motion and θ_i are constant depending on the hysteretic model

Coefficient Method (FEMA 356)

Maximum displacement demand of the nonlinear MDOF system
(target displacement)

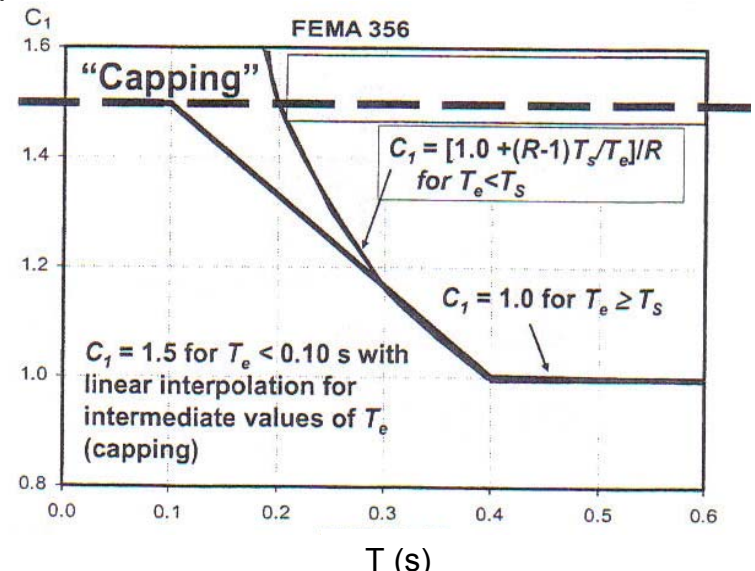
$$\delta_t = C_0 C_1 C_2 C_3 \delta_{SDOFS} = C_0 C_1 C_2 C_3 S_d g$$

C_0 (shape factor): converts the spectral displacement into the control node displacement
(MPF, displacement shape at target displacement or tabulated values)

C_1 (inelastic displacement ratio):

$$\Delta_{NL}/\Delta_e$$

- R- μ -T relationships
- “Capping” for $T < 0.1$ s



C_2 : accounts for pinched hysteresis shape, stiffness degradation and strength deterioration (values for different soil framing system and structural performance levels)

Table 3-3 Values for Modification Factor C_2

Structural Performance Level	$T \leq 0.1 \text{ second}^3$		$T \geq T_s \text{ second}^3$	
	Framing Type 1 ¹	Framing Type 2 ²	Framing Type 1 ¹	Framing Type 2 ²
Immediate Occupancy	1.0	1.0	1.0	1.0
Life Safety	1.3	1.0	1.1	1.0
Collapse Prevention	1.5	1.0	1.2	1.0

1. Structures in which more than 30% of the story shear at any level is resisted by any combination of the following components, elements, or frames: ordinary moment-resisting frames, concentrically-braced frames, frames with partially-restrained connections, tension-only braces, unreinforced masonry walls, shear-critical, piers, and spandrels of reinforced concrete or masonry.
2. All frames not assigned to Framing Type 1.
3. Linear interpolation shall be used for intermediate values of T .

C_3 : accounts for dynamic P- Δ effects

$$C_3 = \begin{cases} 1 + \frac{|\alpha|(R-1)^{3/2}}{T_e} \\ 1 \end{cases} \quad (\text{With positive post elastic stiffness})$$

Coefficient Method (FEMA 356)

Procedure:

- Define the period of the linear SDOF accounting for some loss of stiffness in the transition from the elastic to the inelastic response (secant stiffness at 60% F_y)

$$k_e = \frac{60\%F_y}{d_{60\%F_y}} \quad T_{eff} = T_i \sqrt{k_i / k_e}$$

- Find the displacement demand from the 5% damped spectrum

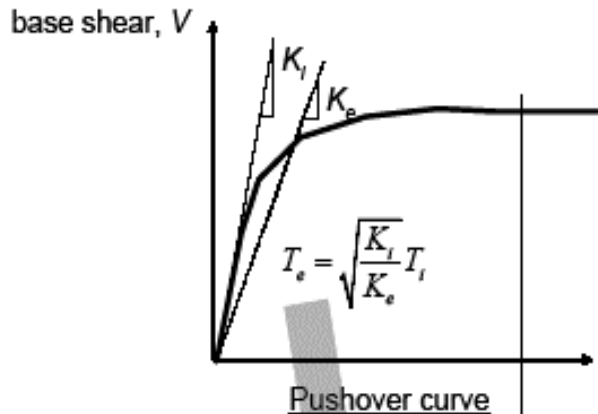
$$S_d = \frac{T_{eff}^2}{4\pi^2} S_a(T_{eff}(F_y), \xi)$$

- S_d depends on T_{eff} , and thus on F_y required

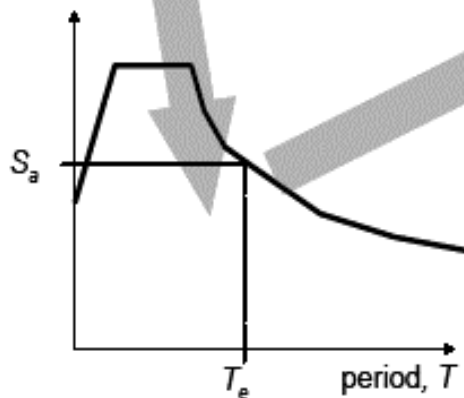
iterative procedure



Coefficient Method (FEMA 356)



$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^2}{4\pi^2} g = \text{Target displacement}$$



- $C_0 =$ converts SDOF spectral displacement to MDOF roof displacement (elastic)
- $C_1 =$ expected maximum inelastic displacement divided by elastic displacement
- $C_2 =$ effects of pinched hysteretic shape, stiffness degradation and strength deterioration
- $C_3 =$ increased displacements due to dynamic P- Δ effects

Coefficient Method (CSM) (FEMA 440 updates)

- The “capping” on C_1 contributes to the inaccuracy of the procedure.

Proposed formulation:

$$C_1 \begin{cases} = 1 + \frac{R-1}{aT^2} & \text{Where: } a = \begin{cases} 130 & \text{for site class B } 760m/s \leq v \leq 1525m/s \\ 90 & \text{for site class C } 360m/s \leq v \leq 760m/s \\ 60 & \text{for site class D } 180m/s \leq v \leq 360m/s \end{cases} \\ = \frac{R-1}{a0.2^2} & \text{For } T < 0.2 \text{ s} \longrightarrow \text{Correct underprediction in this period range} \\ = 1 & \text{For } T > 1 \text{ s} \longrightarrow \text{Equal displacement approximation} \end{cases}$$

- C_2 existing values leads to
 - overestimate the response for $T > 1$ s
 - overestimate, for small R , and underestimate the response for large R , for short period systems
 - only effects due to stiffness degradation is included

$$C_2 \begin{cases} = 1 + \frac{1}{800} \left(\frac{R-1}{T} \right)^2 \\ = 1 + \frac{1}{800} \left(\frac{R-1}{0.2} \right)^2 & \text{For } T < 0.2 \text{ s} \\ = 1 & \text{For } T > 0.7 \text{ s} \end{cases}$$

- C_3 :
 - replaced by the limitation on R (the same of the CSM holds)
 - accounts for the effects of strength degradation

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